

# Adsorption Process Development

## Basics & Methodology

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# Outline

- 🌐 Objectives;
- 🌐 Adsorbents;
- 🌐 Terminology;
- 🌐 Modeling;
- 🌐 Equilibrium theory;
- 🌐 Intraparticle kinetics - LDF model;
- 🌐 Old models;
- 🌐 Methodology;
- 🌐 PSA and SMB technologies.

# Objectives

- 1) Purification
- 2) Recovery of solutes
- 3) Separations (bulk)

## Examples

- a. Phenolic waste waters
- b. Recovery of antibiotics from fermentation broths
- c. Parex/Sarex
- d. Separation of enantiomeres
- e. O<sub>2</sub> from air
- f. N<sub>2</sub> from air
- g. H<sub>2</sub> purification
- h. Propane/propene
- i. Landfill gas

# Adsorbents

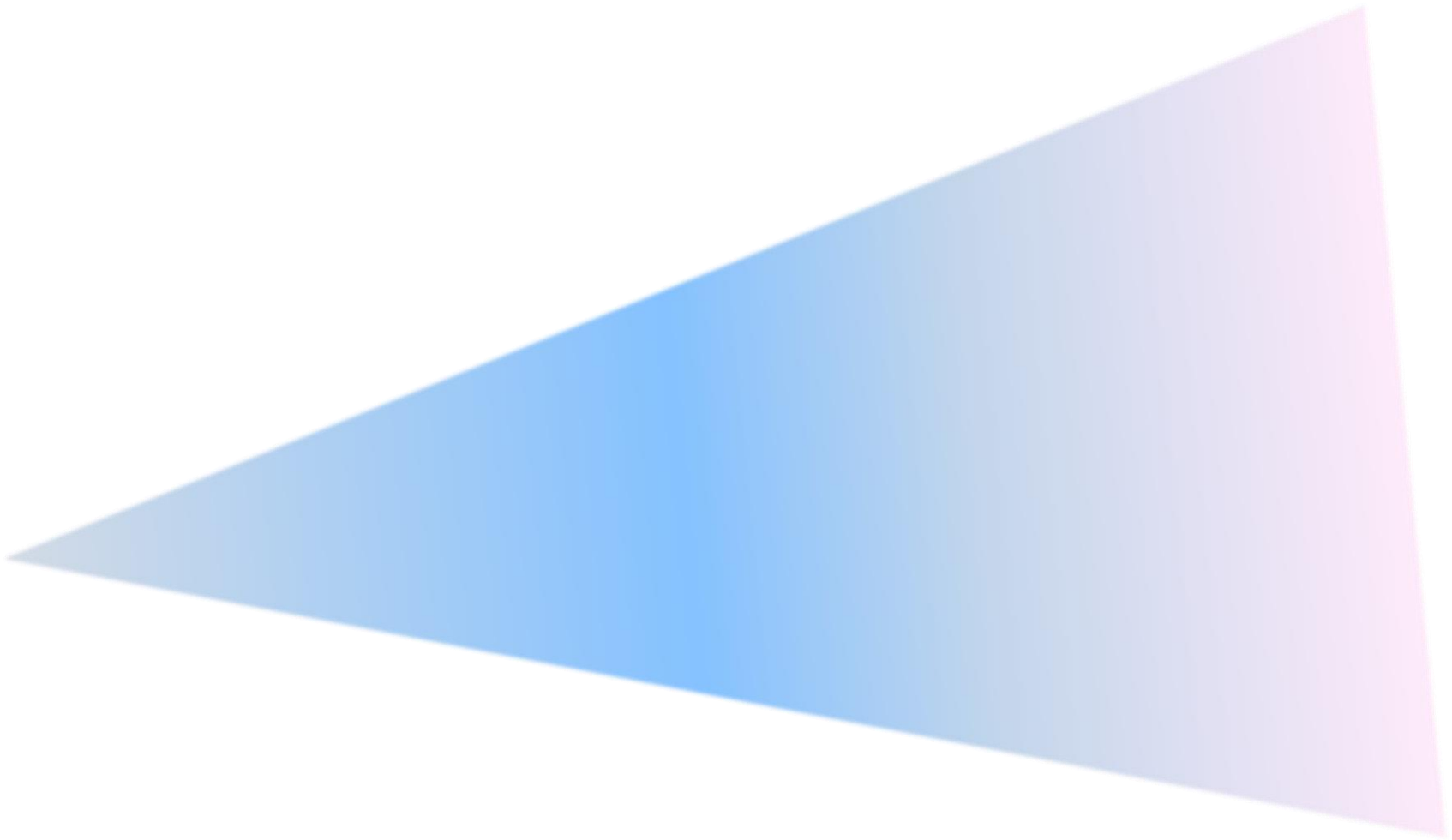
## Types

- Activated carbons
- Carbon molecular sieves
- Zeolites
- Polymers
- Metal organic frameworks (MOFs)

## Structures

- Homogeneous
- Porous
- Bidisperse

# Adsorbents: properties

- Adsorption capacity;
  - Adsorption kinetics;
  - Selectivity;
  - Mechanical stability;
  - Thermal stability;
  - Chemical stability.
- 

Adsorption capacity: Amount of each species that is retained by the adsorbent

Adsorption kinetics: rate at which each species is retained by the adsorbent

**Equilibrium selectivity** – ratio between the adsorption capacities of the more adsorbed species and the less adsorbed species;

Selectivity:

**Kinetic selectivity** – ratio between the kinetic parameter of the more adsorbed species and the less adsorbed species;

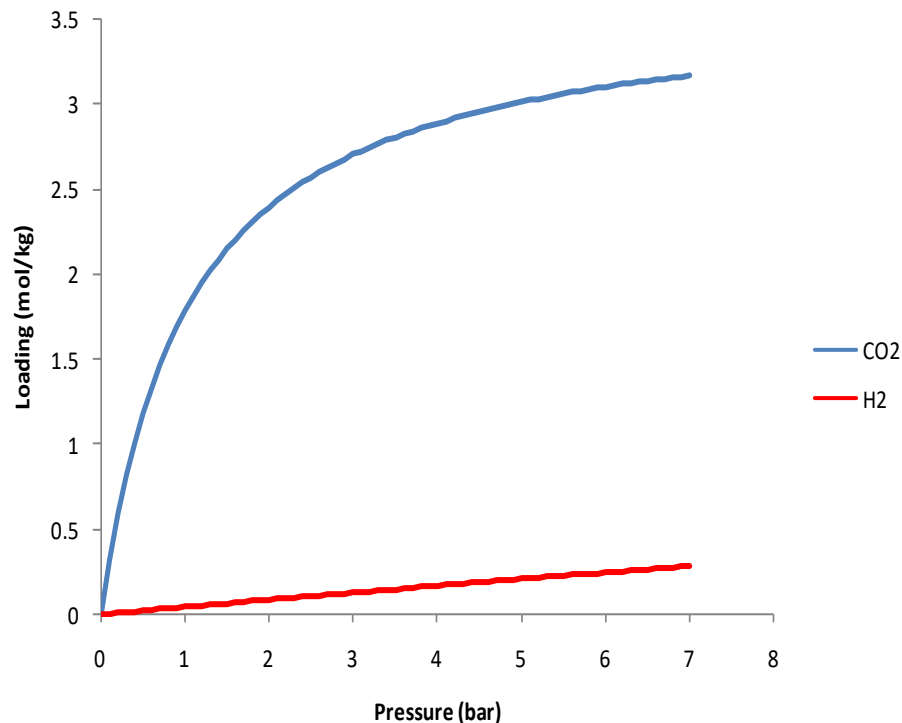
Mechanical stability: Resistance of the adsorbent – the formation of dust may damage the valves and contaminate the product.  
It also limits the height of the columns.

Thermal stability: It is important to know how the adsorbent reacts to temperature increase.  
Some adsorbent can be damaged when increasing the temperature.

Chemical stability: Some adsorbents can be damaged in presence of some species such as water vapour or ammonia.  
It is important to know if these species are in the feed streams and if so, take measures to avoid the degradation of the adsorbent.

# Equilibrium-controlled separation

## Adsorption capacity of H<sub>2</sub> and CO<sub>2</sub> in activated carbon



There is a significant difference between the H<sub>2</sub> and CO<sub>2</sub> adsorption capacities.

This difference in the adsorption capacity allows the separation of these two gases.

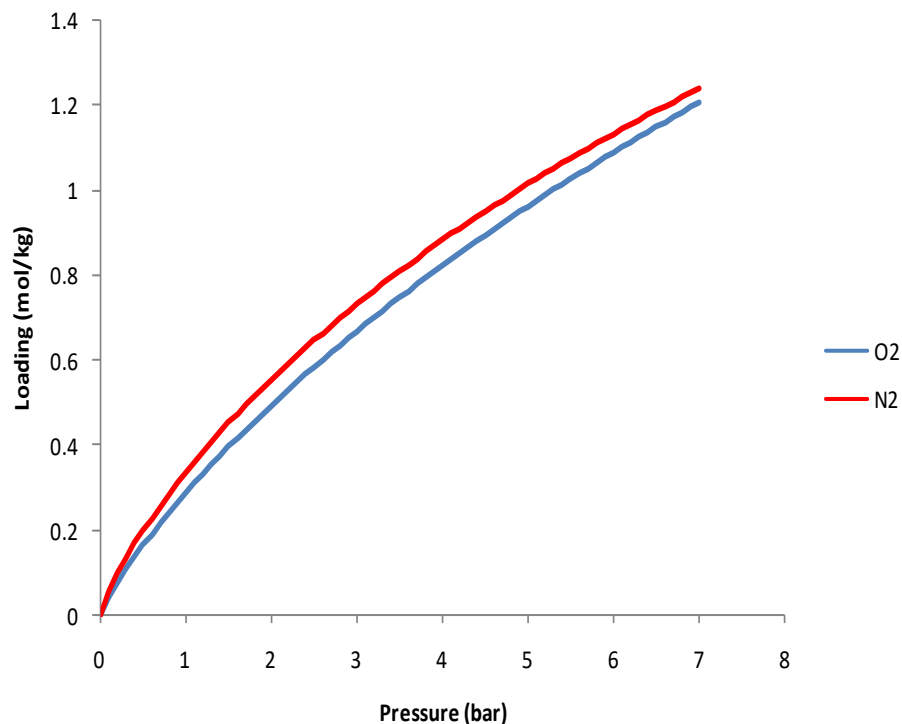
This is an equilibrium-controlled separation.

Sircar, S.; Golden, T. C. Isothermal and Isobaric Desorption of Carbon-Dioxide by Purge. *Ind Eng Chem Res* **1995**, *34*, 2881.



# Kinetic-controlled separation

## Adsorption capacity of N<sub>2</sub> and O<sub>2</sub> in a carbon molecular sieve



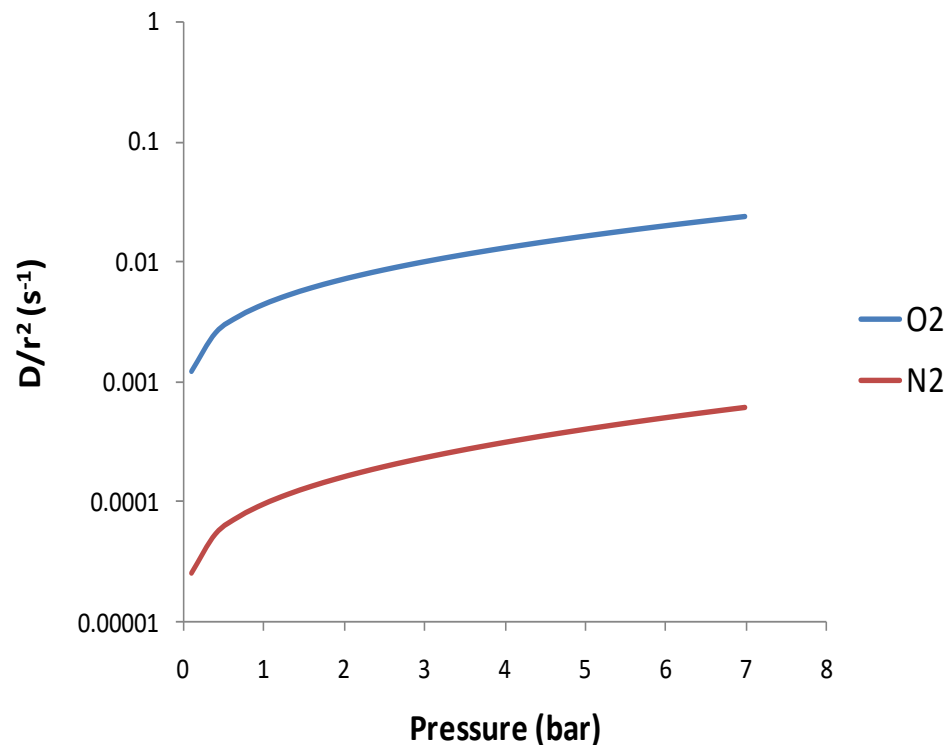
Carbon molecular sieves are commonly used for nitrogen production from air by PSA.

How can the separation be possible if both gases adsorb almost the same?

Jee, J. G.; Kim, M. B.; Lee, C. H. Pressure Swing Adsorption Processes to Purify Oxygen Using a Carbon Molecular Sieve. *Chem Eng Sci*, 60, 869, 2005.

# Kinetic-controlled separation

## Adsorption kinetics of N<sub>2</sub> and O<sub>2</sub> in a carbon molecular sieve



Oxygen diffuses faster than nitrogen.

This difference in the adsorption kinetics allows the separation of these two gases.

This is a kinetic-controlled separation.

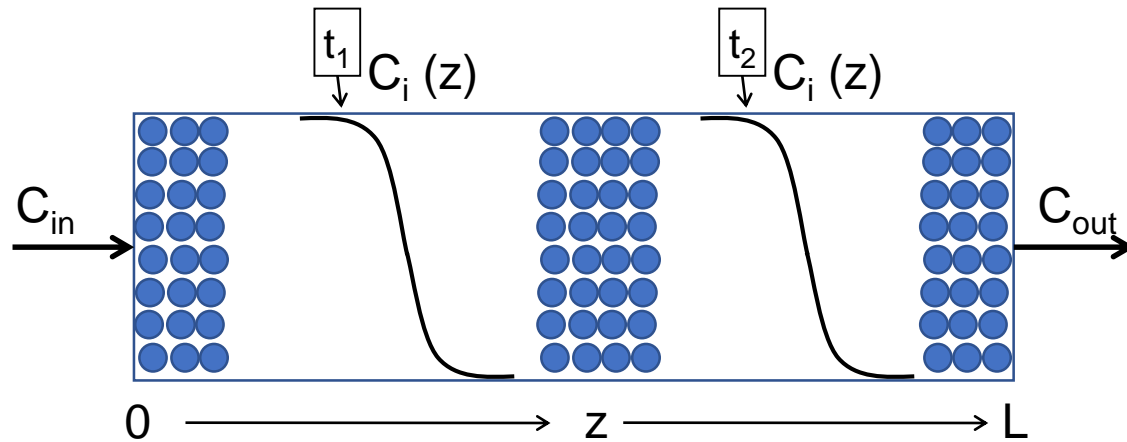
Jee, J. G.; Kim, M. B.; Lee, C. H. Pressure Swing Adsorption Processes to Purify Oxygen Using a Carbon Molecular Sieve. *Chem Eng Sci*, 60, 869, 2005.

# Modelling, Simulation and Optimization

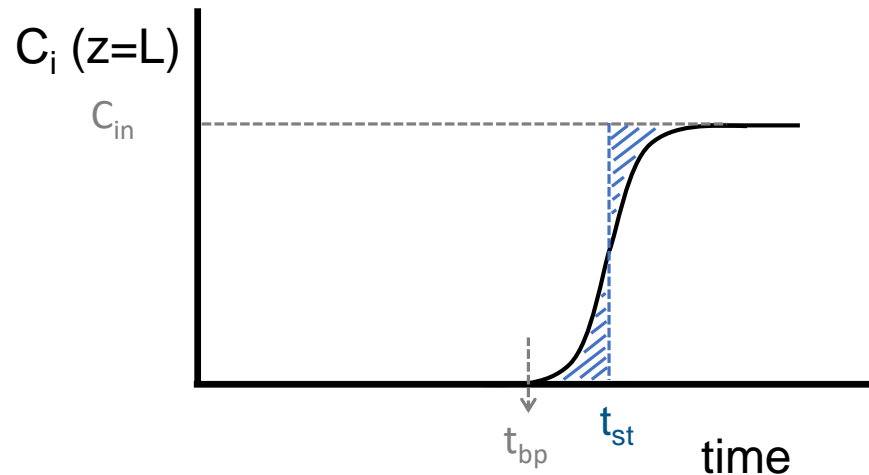
An accurate process simulator is an important tool for learning, designing and optimization purposes.

# Terminology

Concentration profiles -  $C_i(z)$  at a given  $t$



Concentration histories -  $C_i(t)$  at a given  $z$



At  $z=L$  Breakthrough Curve

$t_{bp}$  – breakthrough time  
 $t_{st}$  – stoichiometric time

# Terminology

## Overall balance

$$\underbrace{Q C_{i0} t_{st}} = \varepsilon C_{i0} V + (1 - \varepsilon) q_{i0} V$$

Moles introduced in the column

$$t_{st} = \tau (1 + \xi)$$

$$\tau = \frac{\varepsilon V}{Q} \quad \text{Space time}$$

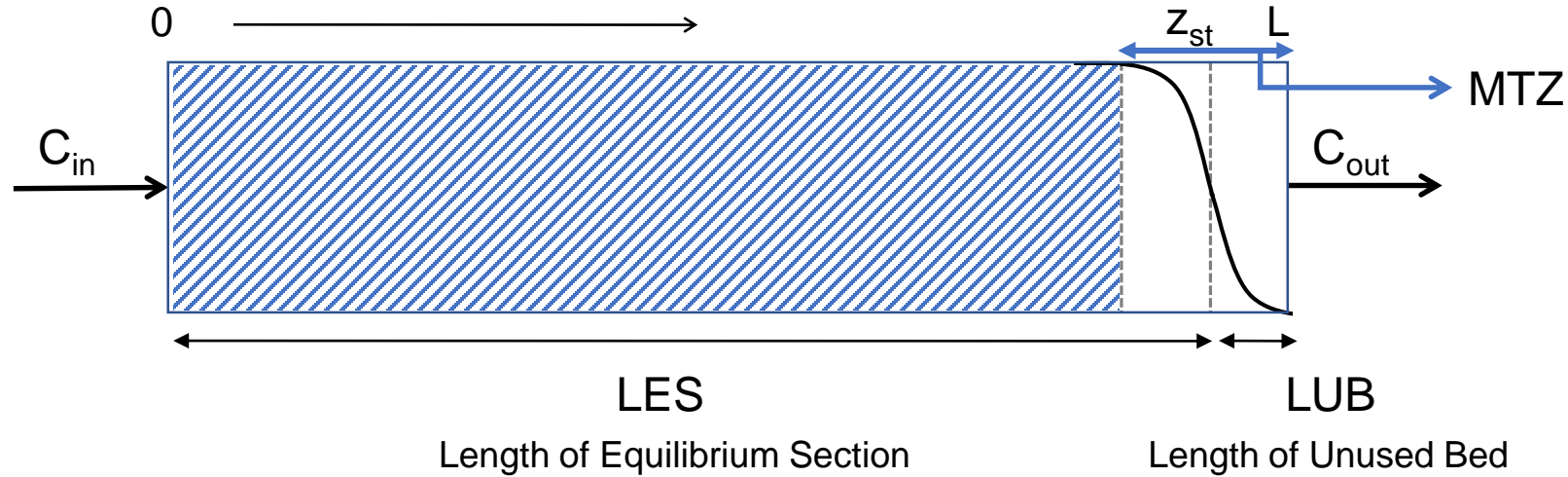
$$\xi = \frac{(1 - \varepsilon)}{\varepsilon} \frac{q_{i0}}{C_{i0}} \quad \text{Capacity factor}$$

$$\text{Total capacity} = Q \int_0^{t_f} (C_{i0} - C_i) dt$$

$$\text{Useful capacity} = Q \int_0^{t_{bp}} (C_{i0} - C_i) dt$$

# Terminology

Concentration profile at  $t = t_{bt}$



$$L = LES + LUB$$

$$L = \frac{u_i}{1 + \xi} t_{st}$$

$$z_{st} = LES = \frac{u_i}{1 + \xi} t_{bp}$$

$$LUB = L - LES = L - \frac{L}{t_{st}} t_{bp}$$

$$LUB = L \left( 1 - \frac{t_{bp}}{t_{st}} \right)$$

$$MTZ = 2LUB$$

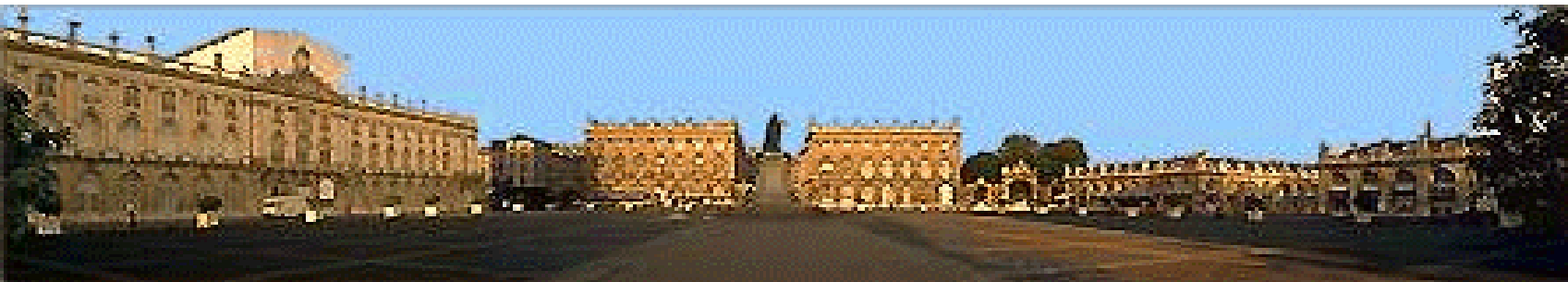
# “Le Génie Chimique c’est pas de la plomberie”

## Modelling

Pierre Le Goff

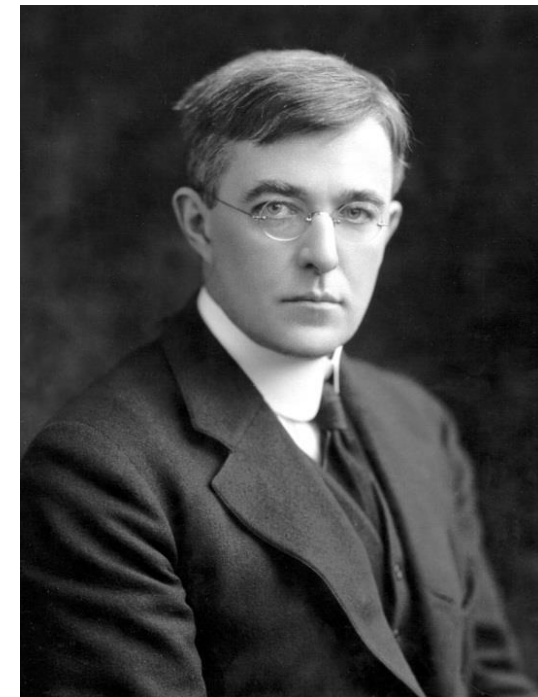
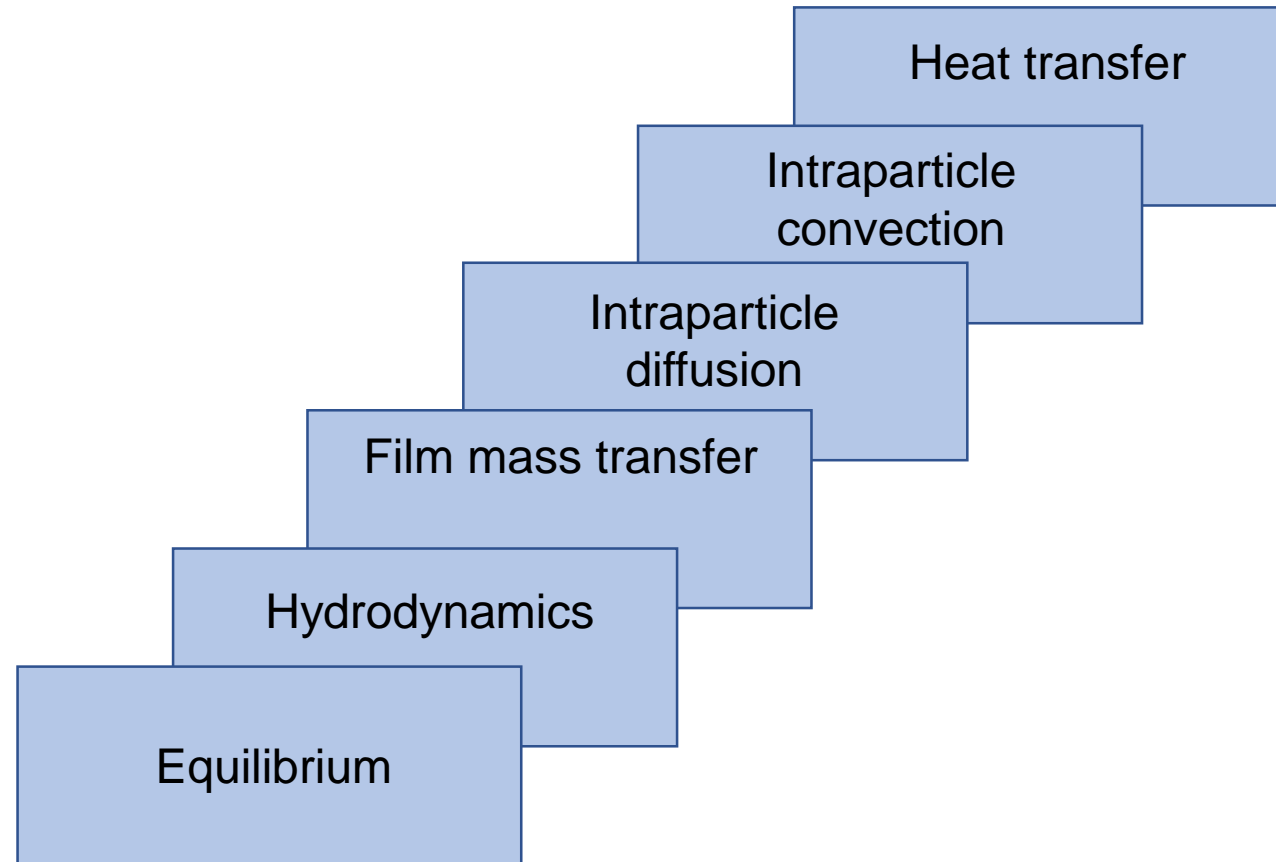


- a) conservation equations (mass, energy, momentum, electric charge)
- b) equilibrium laws at the interface(s)
- c) constitutive laws
- d) kinetic laws of heat/mass transfer and reaction
- e) initial and boundary conditions
- f) optimization criterion



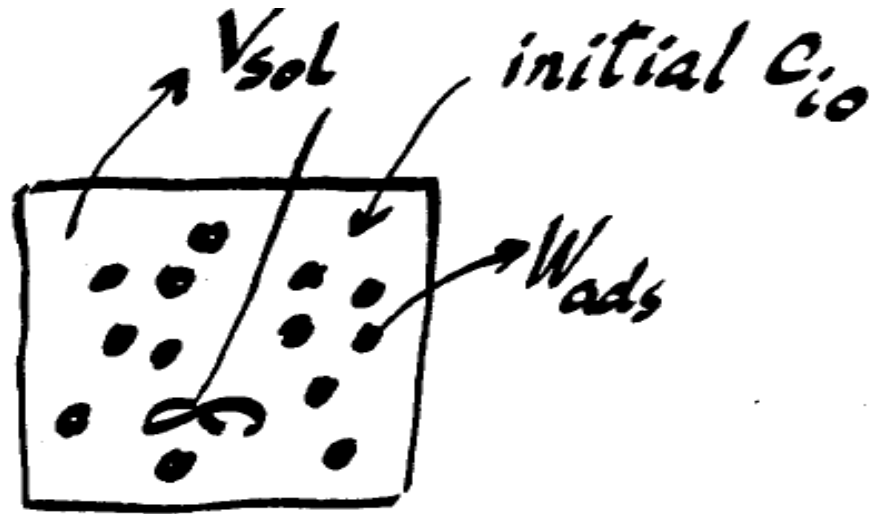
# Modelling

## Factors influencing the behavior of adsorptive processes





# Batch adsorption (isothermal)



Operating Line

Mass balance in a batch adsorber

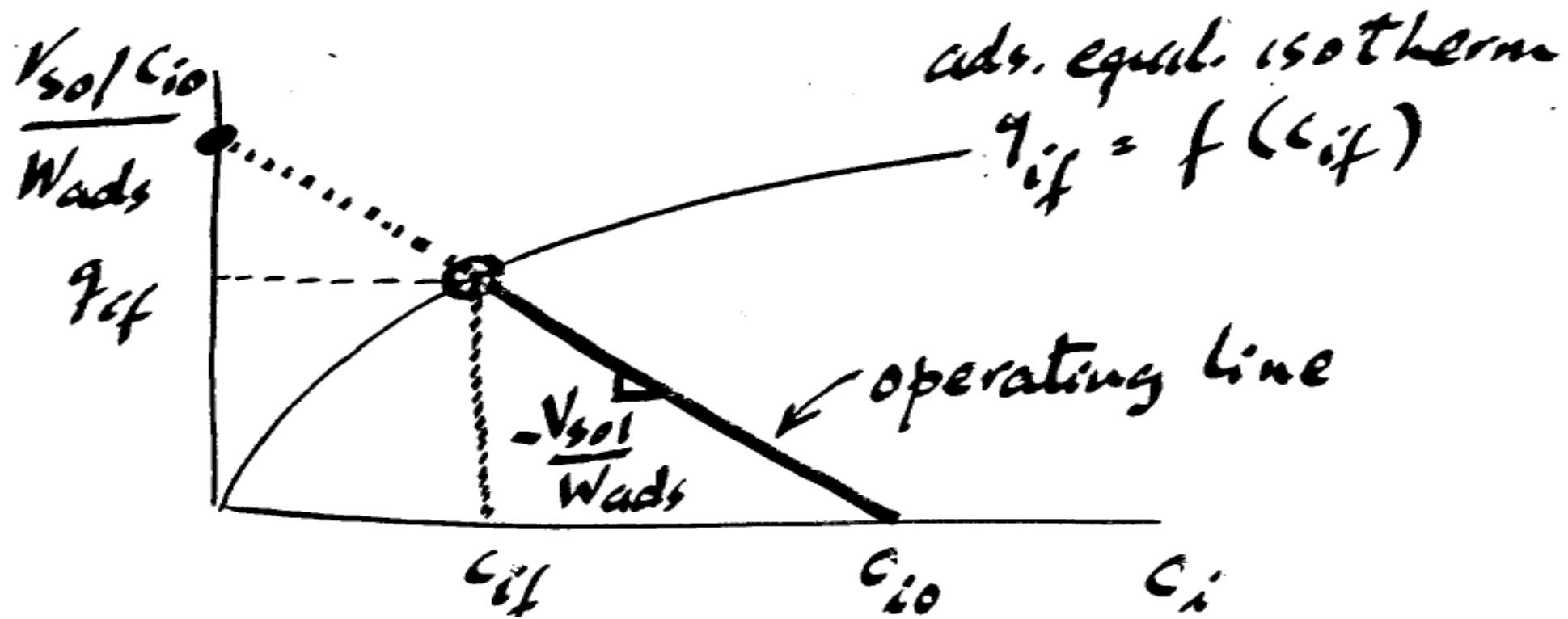
$$-V \frac{dC_i}{dt} = W \frac{dq_i}{dt}$$

$$q_i(t) = \frac{V}{W} C_{i0} - \frac{V}{W} C_i$$

At equilibrium:

$$q_{ie} = \frac{V}{W} C_{i0} - \frac{V}{W} C_{ie}$$

$$q_{ie} = f(C_{ie})$$



Solve

$$\begin{cases} q_{if} = f(C_{if}) \\ q_{if} = \frac{V_{sol} C_{i0}}{W_{ads}} - \frac{V_{sol}}{W_{ads}} C_{if} \end{cases} \rightarrow C_{if}$$

Linear isotherm

$$C_{if} = \frac{C_{i0}}{1 + \xi_m}$$

Factor de capacidade

$$\xi = \frac{(1 - \varepsilon)}{\varepsilon} \frac{q_{i0}}{C_{i0}}$$

Rectangular

$$q_{if} = Q \quad \text{for } C_{if} > 0$$

$$C_{if} = C_{i0} (1 - \xi_m)$$

Langmuir

$$q_{if} = \frac{Q K C_{if}}{1 + K C_{if}}$$

$$x_{if} = \frac{-[1 + K'(\xi_m - 1)] + \sqrt{[1 + K'(\xi_m - 1)]^2 + 4K'}}{2K'}$$

$$K' = K C_{i0}$$

$$x_{if} = C_{if}/C_{i0}$$

# Kinetics of batch adsorption

Mass balance

$$-V \frac{dC_i}{dt} = W \frac{dq_i}{dt} \qquad q_i = \frac{V(C_{i0} - C_i)}{W}$$

Kinetic law

$$\frac{dq_i}{dt} = k_h (q_i^* - q_i) \qquad \text{LDF model}$$

Equilibrium law

$$q_i^* = KC_i \qquad \text{Linear isotherm}$$

Initial condition

$$t = 0; C_i = C_{i0}; q_i = q_{i0}$$

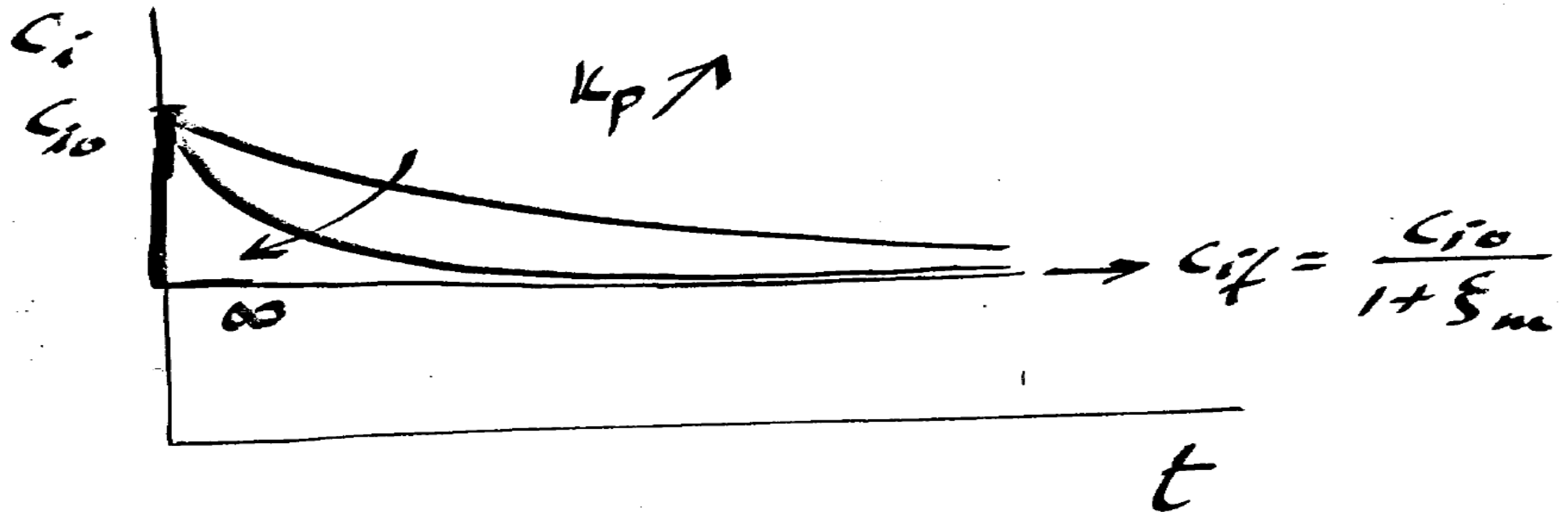


## Linear Driving Force model Glueckauf

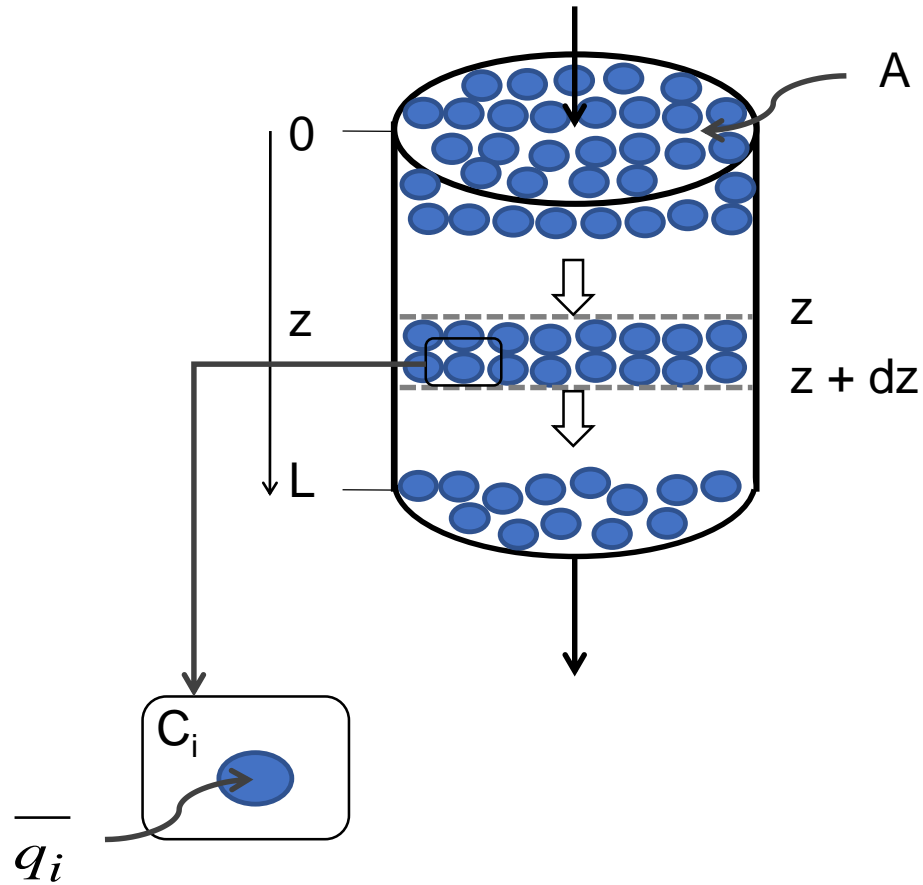
$$\frac{\partial \langle q \rangle}{\partial t} = k_h [q_s - \langle q \rangle]$$

$$k_h = \frac{15 D_h}{R_p^2}$$

$$C_i(t) = \frac{C_{i0}}{1+X} (1 + Xe^{-k_h(1+X)t})$$



# Mass balance for species i



Average adsorbed concentration in the particle (mol/m<sup>3</sup><sub>ads</sub>)

Isothermal operation

Axial dispersed flow

$$\begin{array}{lcl} \text{Flux in} & & (\varepsilon A) \varphi_z \\ \text{mol/s} & & \text{m}^2 \quad \text{mol/m}^2\text{s} \end{array}$$

$$\text{Flux out} \quad (\varepsilon A) \varphi_{z+dz}$$

Acumulation

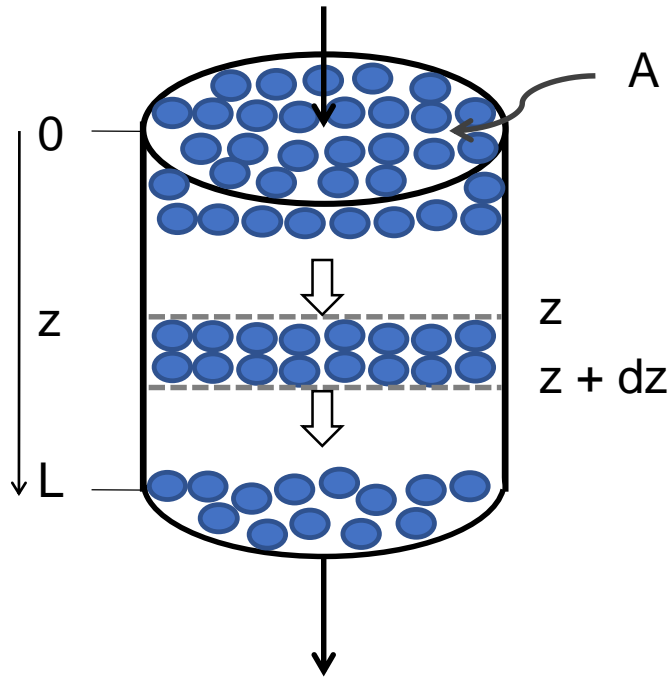
$$\text{Interparticle} \quad \varepsilon A dz \frac{\partial C_i}{\partial t}$$

$$\text{Intraparticle} \quad (1 - \varepsilon) A dz \frac{\partial \bar{q}_i}{\partial t}$$

$$(\varepsilon A) \varphi_z = (\varepsilon A) \varphi_{z+dz} + \varepsilon A dz \frac{\partial C_i}{\partial t} + (1 - \varepsilon) A dz \frac{\partial \bar{q}_i}{\partial t}$$

$$0 = \frac{\partial \varphi_i}{\partial z} + \frac{\partial C_i}{\partial t} + \frac{(1 - \varepsilon)}{\varepsilon} \frac{\partial \bar{q}_i}{\partial t}$$

# Mass balance for species i



Isothermal operation

Axial dispersed flow

$$0 = \frac{\partial \varphi_i}{\partial z} + \frac{\partial C_i}{\partial t} + \frac{(1 - \varepsilon)}{\varepsilon} \frac{\partial \bar{q}_i}{\partial t}$$

$$\varphi = u_i C_i - D_{ax} \frac{\partial C_i}{\partial z}$$

$$u_i = \frac{u_0}{\varepsilon}$$

$$\varepsilon D_{ax} \frac{\partial^2 C_i}{\partial z^2} = u_0 \frac{\partial C_i}{\partial z} + \varepsilon \frac{\partial C_i}{\partial t} + (1 - \varepsilon) \frac{\partial \bar{q}_i}{\partial t}$$



# Mass balance for species i

## Dimensionless variables

$$x = \frac{z}{L}$$

$$\theta = \frac{t}{\lambda}$$

$$\tilde{C}_i = \frac{C_i}{C_{i0}}$$

$$\tilde{q}_i = \frac{q_i}{q_{i0}}$$

$$\frac{1}{Pe} \frac{\partial^2 \tilde{C}_i}{\partial x^2} = \frac{\partial \tilde{C}_i}{\partial x} + \frac{\partial \tilde{C}_i}{\partial \theta} + \xi \frac{\partial \tilde{q}_i}{\partial \theta}$$

Peclet number

$$Pe = \frac{u_0 L}{\varepsilon D_{ax}}$$

$$Pe \rightarrow \infty$$

Plug flow of fluid phase

Capacity factor

$$\xi = \frac{(1 - \varepsilon)}{\varepsilon} \frac{q_{i0}}{C_{i0}}$$

$$\xi \rightarrow 0$$

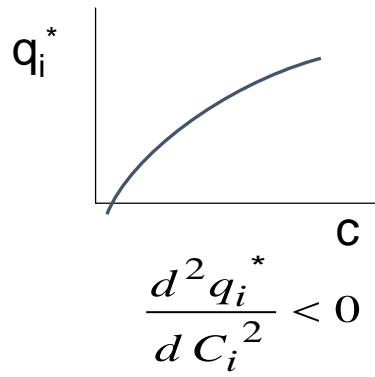
No adsorption, inert packing

# Equilibrium law at interfaces

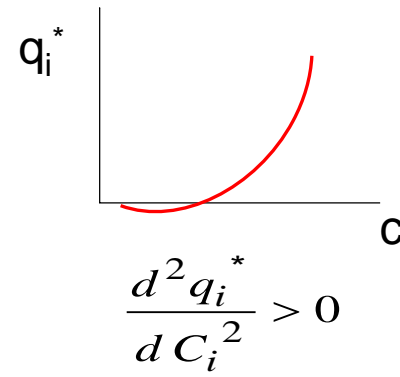
## Adsorption equilibrium isotherm

$$q_i^* = f(C_i)$$

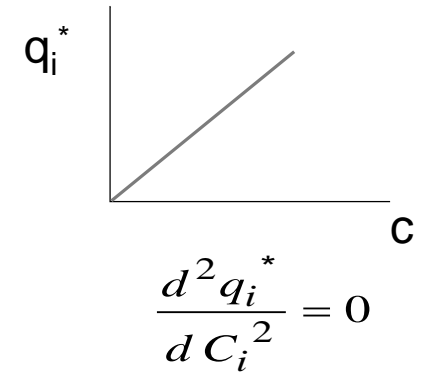
### Favorable isotherms



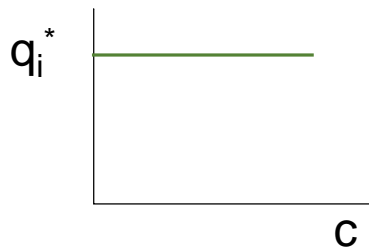
### Unfavorable isotherms



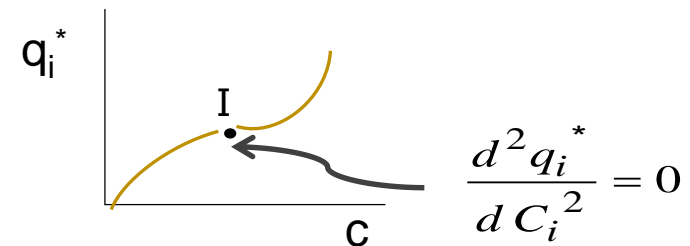
### Linear isotherms



### Rectangular or irreversible



### With an inflection point (BET)



# Equilibrium theory

## The simplest model

- ✓ Isothermal operation
- ✓ Instantaneous equilibrium in each point of the bed -
- ✓ Plug flow
- ✓ No pressure drop (negligible)

$$\overline{q_i} = q_i^*$$

a) Mass balance for i

$$u_0 \frac{\partial C_i}{\partial z} + \varepsilon \frac{\partial C_i}{\partial t} + (1 - \varepsilon) \frac{\partial q_i^*}{\partial t} = 0$$

b) Equilibrium law

$$q_i^* = f(C_i)$$

Combining a) and b)

$$u_i \frac{\partial C_i}{\partial z} + \frac{\partial C_i}{\partial t} + \frac{(1 - \varepsilon)}{\varepsilon} f'(C_i) \frac{\partial C_i}{\partial t} = 0$$

$$u_i \frac{\partial C_i}{\partial z} + \left[ 1 + \frac{(1 - \varepsilon)}{\varepsilon} f'(C_i) \right] \frac{\partial C_i}{\partial t} = 0$$

# Equilibrium theory

$$u_i \frac{\partial C_i}{\partial z} + \left[ 1 + \frac{(1 - \varepsilon)}{\varepsilon} f'(C_i) \right] \frac{\partial C_i}{\partial t} = 0$$

Since

$$\left( \frac{\partial z}{\partial t} \right)_C = - \frac{\left( \frac{\partial C}{\partial t} \right)_z}{\left( \frac{\partial C}{\partial z} \right)_t}$$

It results in:

$$u_c = \left( \frac{\partial z}{\partial t} \right)_C = \frac{u_i}{1 + \frac{1 - \varepsilon}{\varepsilon} f'(C_i)}$$

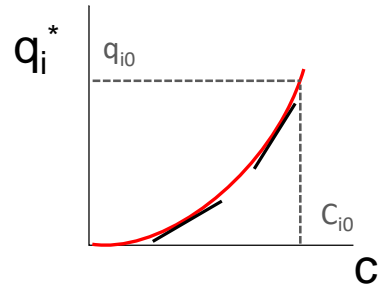
De Vault's equation (1943)

Adsorption as a wave  
phenomenon

The velocity of propagation of a concentration  $C$ , i.e.  $u_c$ , is inversely proportional to the local slope of the isotherm  $f'(C_i)$

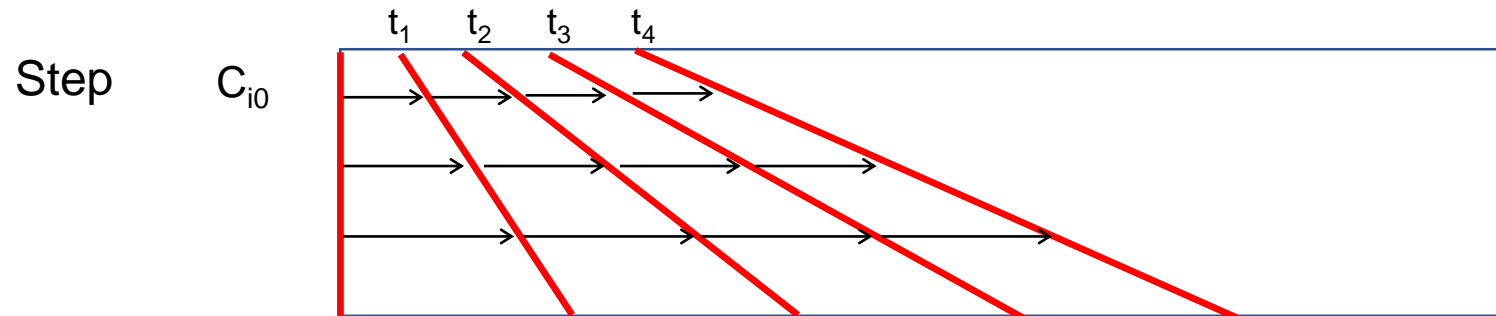
# Equilibrium theory

## Unfavorable isotherms



As  $C_i \nearrow$  the slope  $f'(C_i) \nearrow$  and  $u_c \searrow$

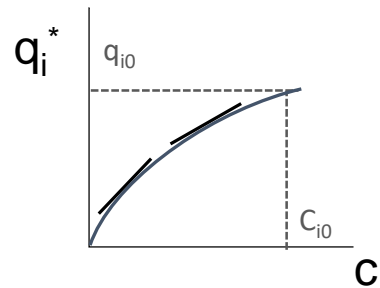
Higher concentrations travel at lower velocities



Concentration profiles -  $C_i(z)$  at a given  $t$

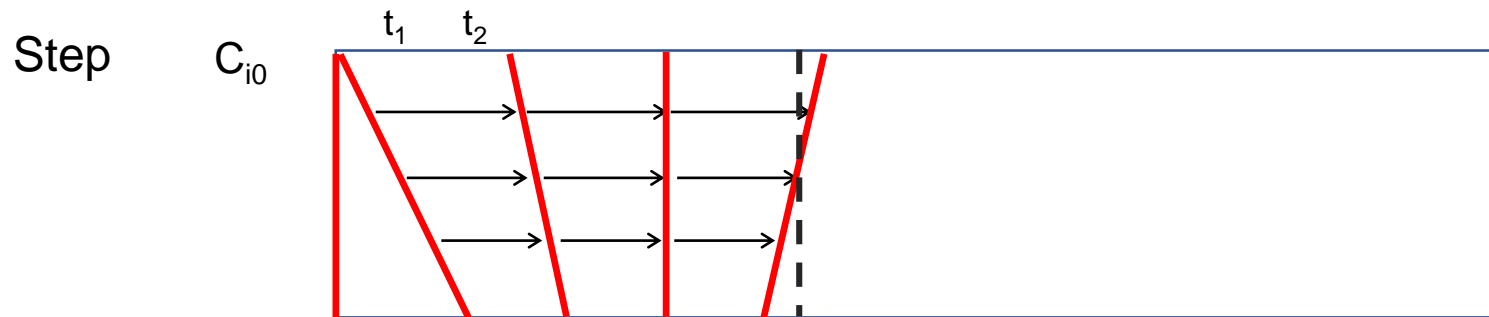
# Equilibrium theory

## Favorable isotherms

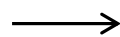


As  $C_i \nearrow$  the slope  $f'(C_i) \searrow$  and  $u_c \nearrow$

Higher concentrations travel at higher velocities



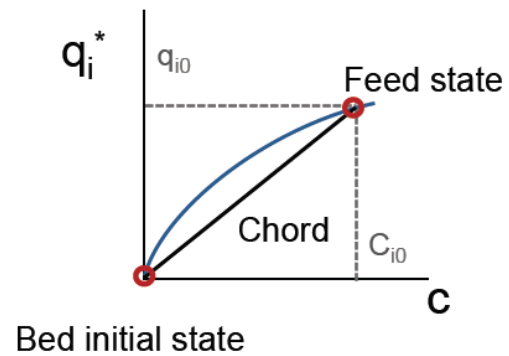
Physically not possible



Compressive front → **shock**

# Equilibrium theory

## Favorable isotherms



## Shock velocity

$$u_{sh} = \frac{u_i}{1 + \frac{1 - \varepsilon}{\varepsilon} \underbrace{\frac{\Delta q_i}{\Delta c_i}}_{\text{Slope of the chord}}}$$

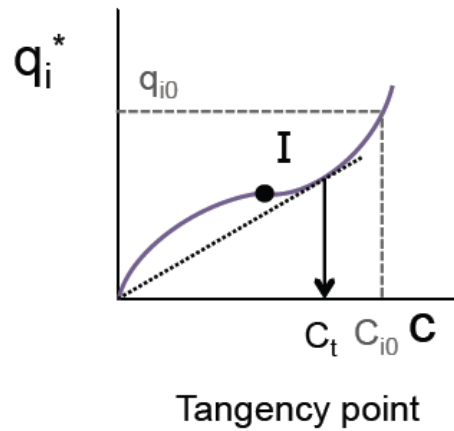
Slope of the chord

# Equilibrium theory

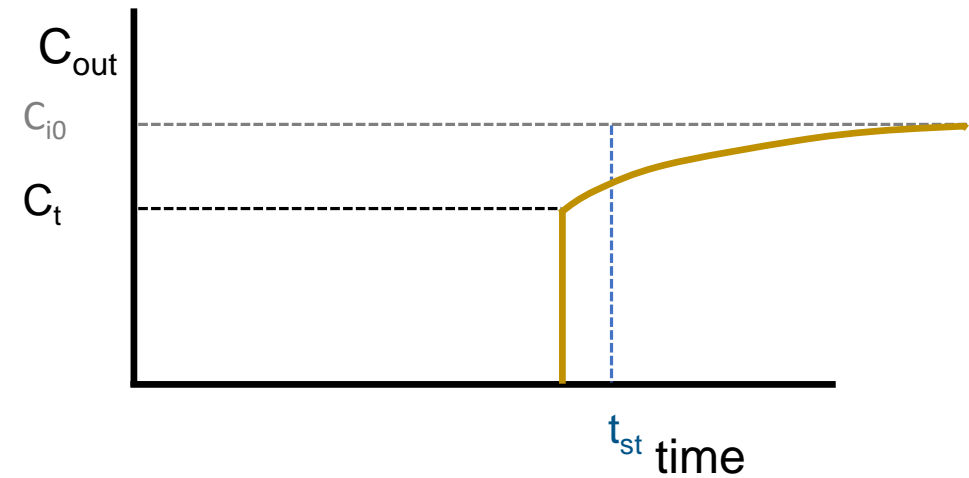
## Linear isotherms

The input is just delayed without change

## BET



Breakthrough curve – feed at  $C_{i0}$



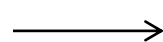
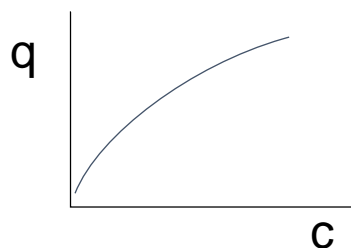
Composite front: shock up to  $C_t$  then dispersive



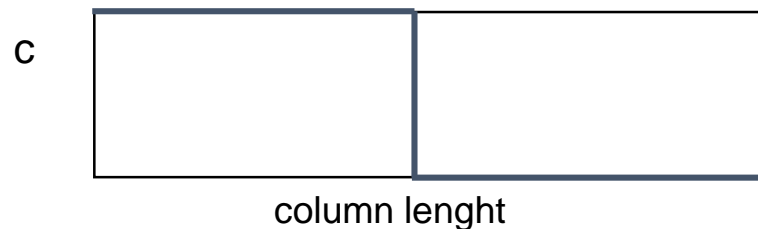
# Equilibrium theory

The nature of the breakthrough curve is governed in first place by the adsorption equilibrium isotherm.

Favorable isotherms

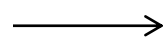
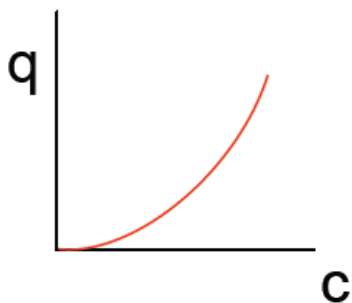


Compressive fronts (shock)

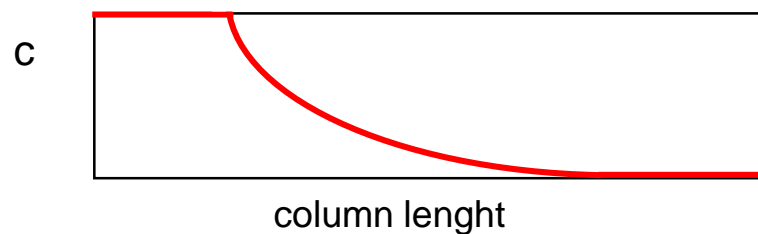


$$u_{sh} = \frac{u_i}{1 + \frac{1 - \varepsilon}{\varepsilon} \frac{\Delta q_i}{\Delta c_i}}$$

Unfavorable isotherms



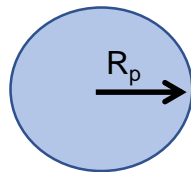
Dispersive fronts



$$u_{c_i} = \frac{u_i}{1 + \frac{1 - \varepsilon}{\varepsilon} f'(c)}$$

# Intraparticle kinetics

## Homogeneous particle



$$\frac{\partial q}{\partial t} = D_h \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial q}{\partial r} \right)$$

Parabolic profile

$$q = a_0 + a_2 r^2$$

$$\frac{\partial q}{\partial r} = 2 a_2 r$$

$$q_s = a_0 + a_2 R_p^2$$

$$\left. \frac{\partial q}{\partial r} \right|_{R_p} = 2 a_2 R_p$$

$$\langle q \rangle = a_0 + a_2 \frac{3}{5} R_p^2$$

Averaging

$$\frac{R_p^3}{3} \frac{\partial \langle q \rangle}{\partial t} = D_h \left[ r^2 \frac{\partial q}{\partial r} \right]_0^{R_p} = D_h R_p^2 \left. \frac{\partial q}{\partial r} \right|_{R_p}$$

$$\frac{\partial \langle q \rangle}{\partial t} = D_h \frac{3}{R_p} 2 a_2 R_p = 6 a_2 D_h$$

## Linear driving force – LDF (Glueckauf)

$$\frac{\partial \langle q \rangle}{\partial t} = k_h [q_s - \langle q \rangle]$$

$$k_h = \frac{15 D_h}{R_p^2}$$

$$6 a_2 D_h = k_h \left[ a_0 + a_2 R_p^2 - a_0 + a_2 \frac{3}{5} R_p^2 \right]$$

# Intraparticle kinetics

## Porous particle

$$\varepsilon_p \frac{\partial C_p}{\partial t} + \frac{\partial q}{\partial t} = D_{pe} \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C_p}{\partial r} \right)$$

$$q = f(C_p)$$

↑  
Per volume of particle

Averaging

$$\varepsilon_p \frac{\partial \langle C_p \rangle}{\partial t} + \frac{\partial \langle q \rangle}{\partial t} = k_p (C_{ps} - \langle C_p \rangle)$$

$$k_p = \frac{15 D_{pe}}{R_p^2}$$

Linear isotherms

$$q = K C_p \quad \frac{\partial \langle C_p \rangle}{\partial t} = \frac{k_p}{\varepsilon + K} (C_{ps} - \langle C_p \rangle)$$

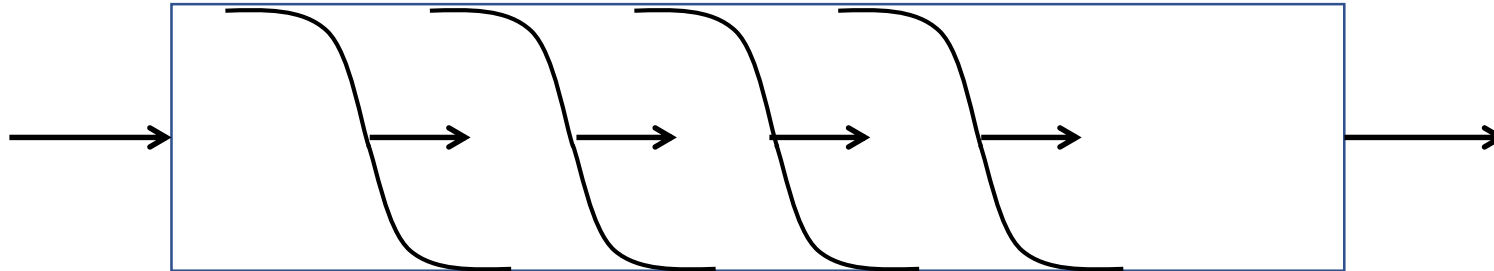
Equivalence with homogeneous particles

$$\frac{\partial \langle q \rangle}{\partial t} = k_h [q_s - \langle q \rangle]$$

$$k_h = \frac{k_p}{K_h} = \frac{k_p}{\varepsilon_p + K}$$

# Constant pattern

## Stationary front



$$C_i(z + dz, t + dt) = C_i(z, t)$$

$$\left. \frac{\partial C_i}{\partial z} \right|_t = -\frac{1}{\lambda} \left. \frac{\partial C_i}{\partial t} \right|_z$$

$$\lambda = \left. \frac{\partial z}{\partial t} \right|_{C_i} = \frac{L}{t_{st}} = \frac{u_i}{1 + \xi}$$

Velocity of the stationary front

# Old models

- I) Chemical kinetic type
- II) Physical kinetic type

# Old models

## Chemical kinetic type – Thomas model

Mass balance

$$u_0 \frac{\partial C_i}{\partial z} + \varepsilon \frac{\partial C_i}{\partial t} + (1 - \varepsilon) \frac{\partial \bar{q}_i}{\partial t} = 0$$

Kinetic law

$$\frac{\partial \bar{q}_i}{\partial t} = k_1 \left[ C (Q - \bar{q}_i) - \frac{1}{K} \bar{q}_i (C_0 - C_i) \right]$$

Boundary and initial conditions

$$\begin{aligned} z = 0 \quad C &= C_0 \quad \forall t \\ t = \frac{z}{u_i} \quad q_i &= 0 \quad \forall z \end{aligned}$$

$$\frac{C_i}{C_0} = \frac{J(rN, NT)}{J(rN, NT) + [1 - J(N, rNT)] \times e^{(r-1)N(T-1)}}$$

$$r = \frac{1}{K}, \quad N = k_1 Q \frac{z}{u_0} (1 - \varepsilon), \quad NT = k_1 C_0 \left( t - \frac{z}{u_i} \right)$$

# Old models

## Chemical kinetic type – Simplifications

$$r = 0 \quad K \rightarrow \infty$$

Bohart

$$\frac{C_i}{C_0} = \frac{e^{NT}}{e^{NT} + e^N - 1}$$

$$r = 1$$

Walter

$$\frac{C_i}{C_0} = J(N, NT)$$

*high r*

(Unfavorable isotherm  $K < 1$ )

$$\frac{C_i}{C_0} = \frac{\sqrt{\frac{r}{T}} - r}{1 - r}$$

$r = 1$ ,  $N$  and  $NT$  high

Klinkenberg

$$\frac{C_i}{C_0} = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \sqrt{NT} - \sqrt{N} \right) \right]$$

# Old models

## Physical kinetic type – Rosen model

Mass balance 
$$u_0 \frac{\partial C_i}{\partial z} + \varepsilon \frac{\partial C_i}{\partial t} + (1 - \varepsilon) \frac{\partial \bar{q}_i}{\partial t} = 0$$

Kinetic law for film mass transfer 
$$\frac{\partial \bar{q}_i}{\partial t} = k_f a (C_i - C_i^s)$$

Intraparticle diffusion 
$$\frac{\partial q_i(r, z, t)}{\partial t} = D_h \left[ \frac{\partial^2 q_i}{\partial r^2} + \frac{2}{r} \frac{\partial q_i}{\partial r} \right]$$

Average 
$$\bar{q}_i(z, t) = \frac{3}{R_p} \int_0^{R_p} q_i(r, z, t) r^2 dr$$

Isotherm 
$$q_i = \frac{Q}{C_0} C_i = m C_i$$

$$\frac{C_i}{C_0} = \frac{1}{2} + \frac{2}{\pi} \int_0^\infty e^{-A \lambda} \sin B \frac{d\lambda}{\lambda}$$

A and B depend on the model parameters

$$Bi = \frac{k_f R_p}{D_h}, \quad N_f = k_f a \tau, \quad m$$



# Methodology

Prior to any simulation, necessary data must be gathered independently:

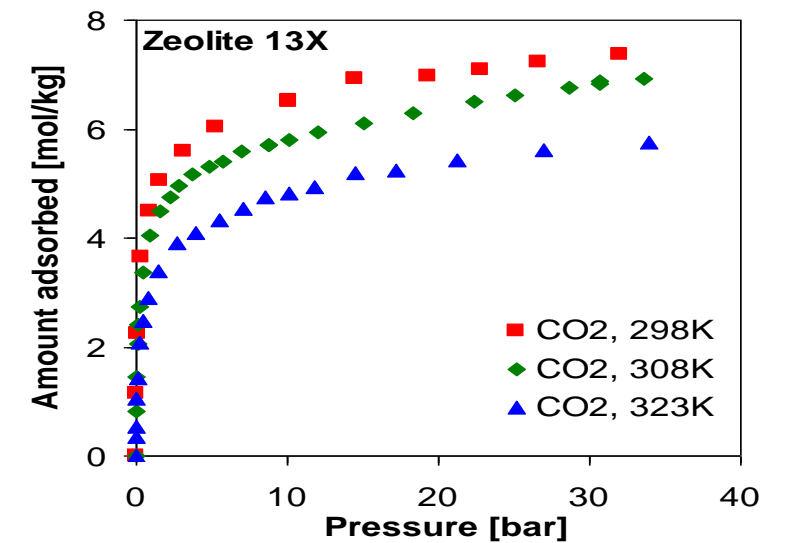
- I. measurement of single adsorption isotherms;
- II. measurement of intraparticle diffusivities (ZLC);
- III. measurement of film mass transfer (shallow bed);
- IV. measurement of axial dispersion (tracer);
- V. measurement of breakthrough curves for single and feed mixtures;
- VI. modeling breakthrough curves and model validation;
- VII. modeling/simulation of process (PSA; SMB);
- VIII. lab-scale operation of process and model validation;
- IX. sizing and scale-up of industrial process.

# Methodology

Single adsorption isotherms - Liquid phase in batch systems

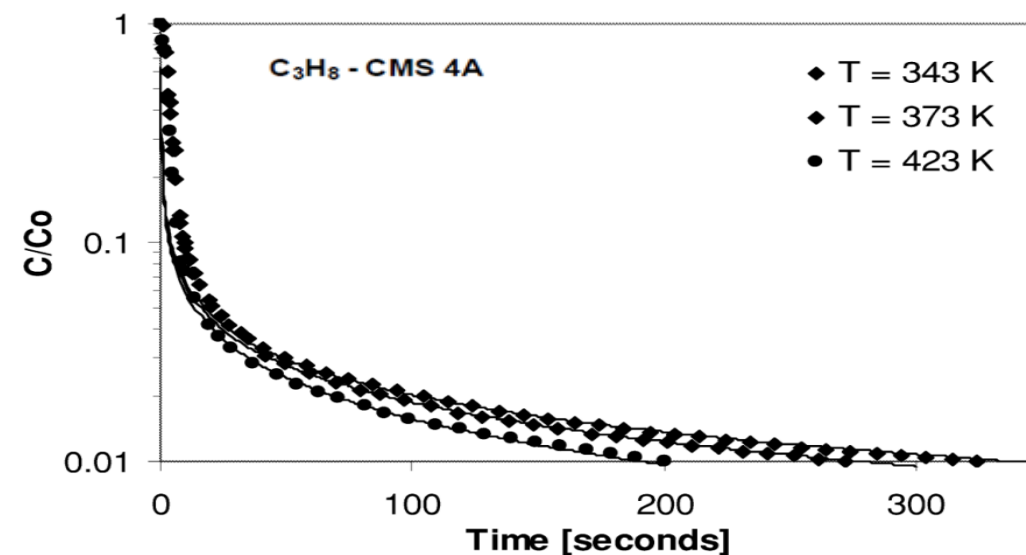
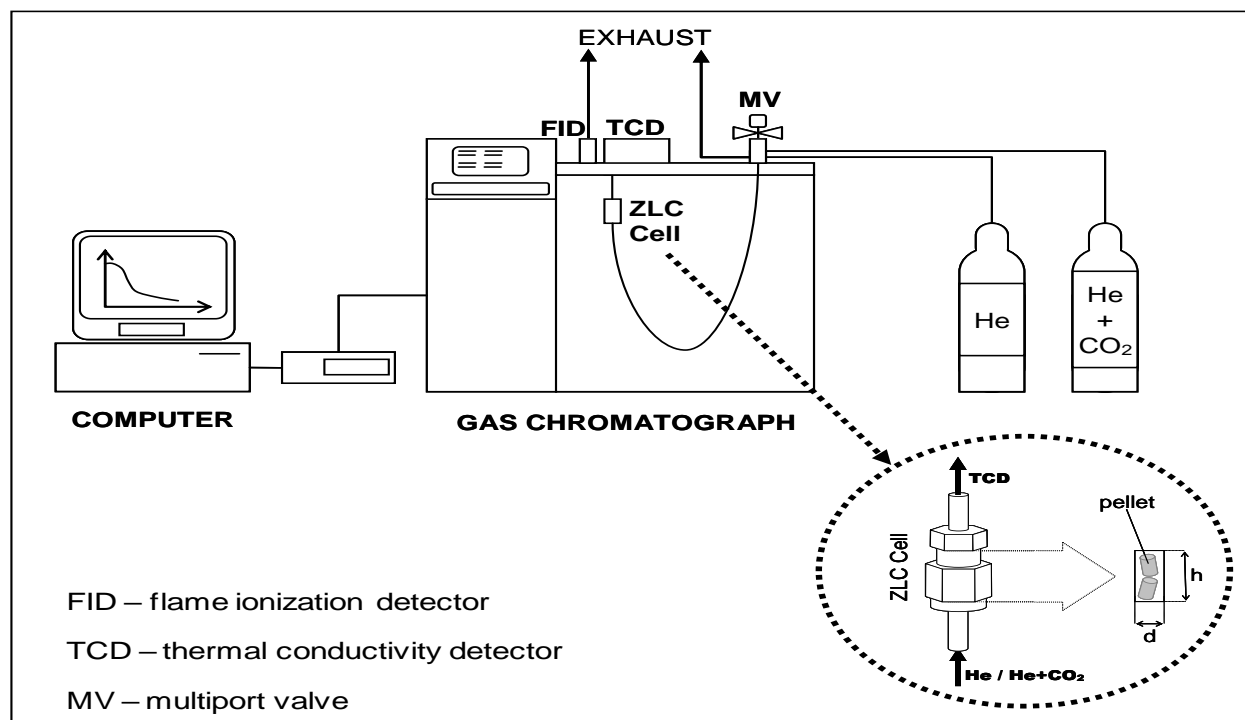
# Methodology

Single adsorption isotherms - Gas phase in Rubotherm magnetic balance



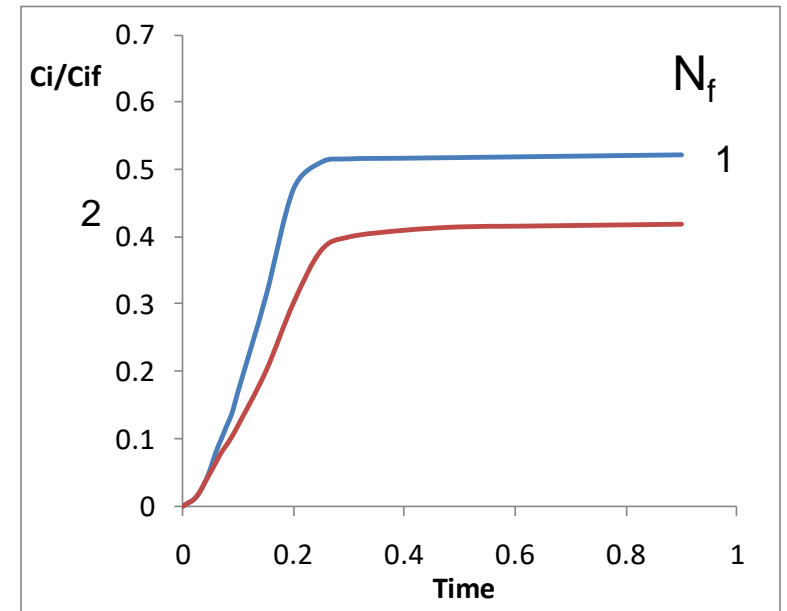
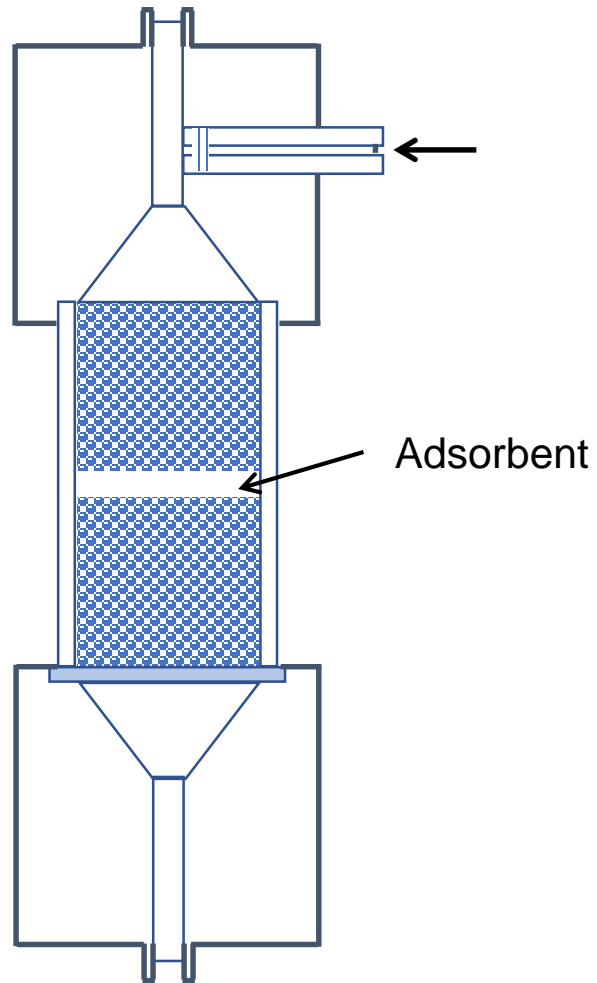
# Methodology

## Intraparticle diffusivities - Zero-length column (ZLC) method



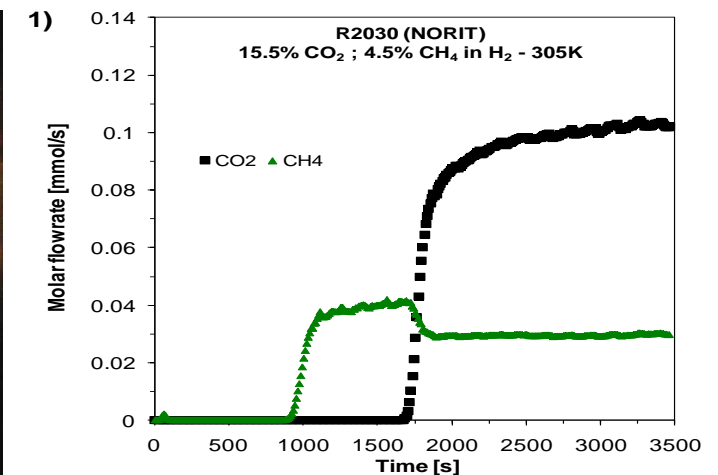
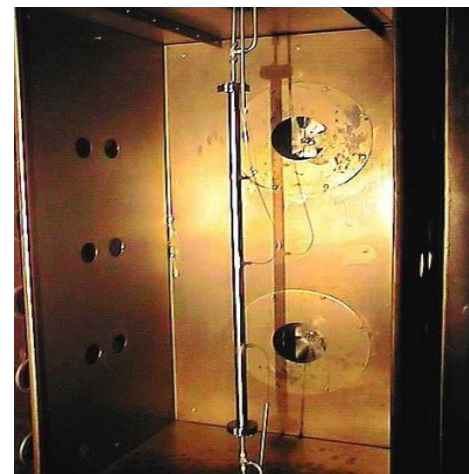
# Methodology

## Film mass transfer - shallow-bed technique

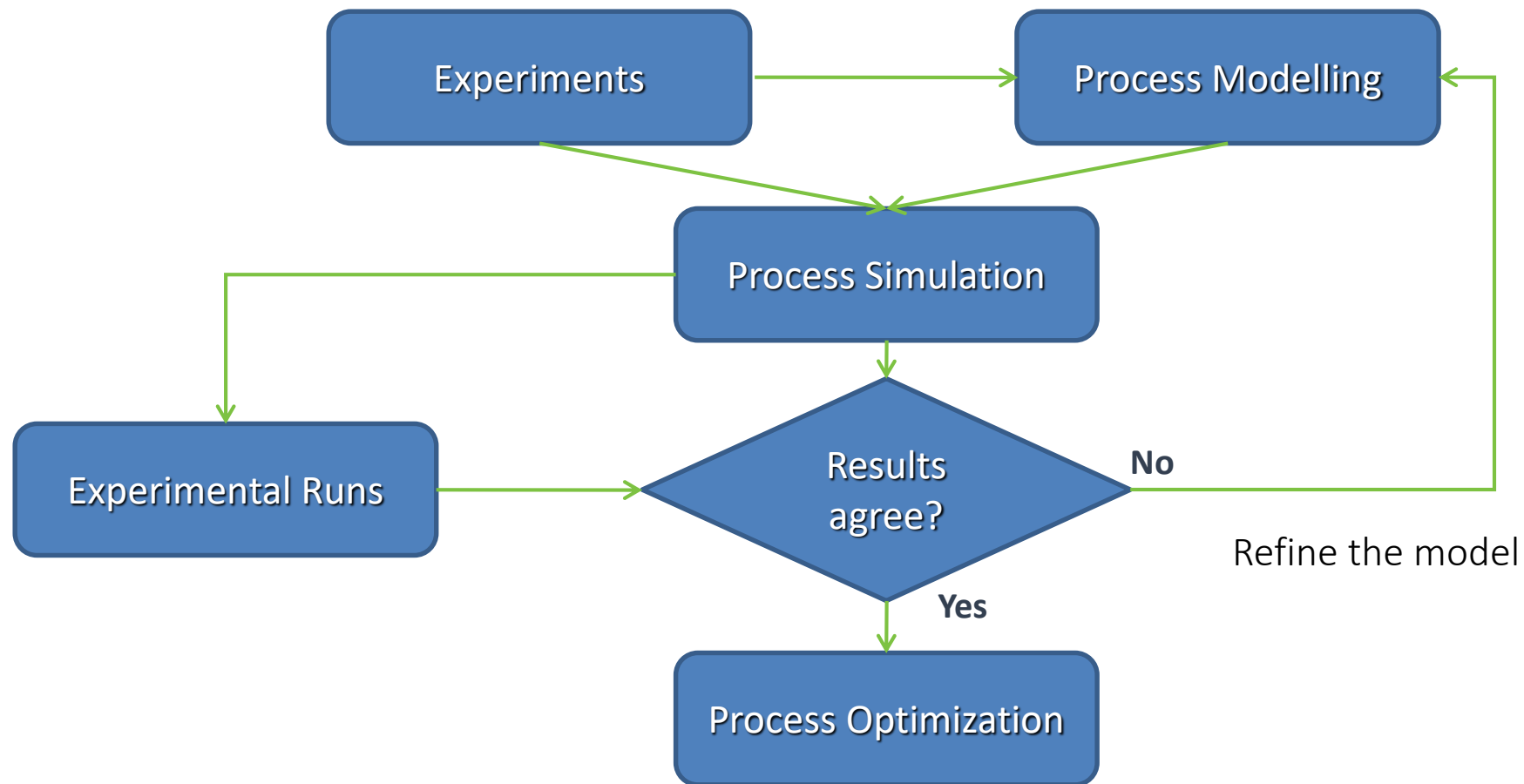


# Methodology

Axial dispersion / model validation - tracer experiments and breakthrough curves for single and feed mixtures

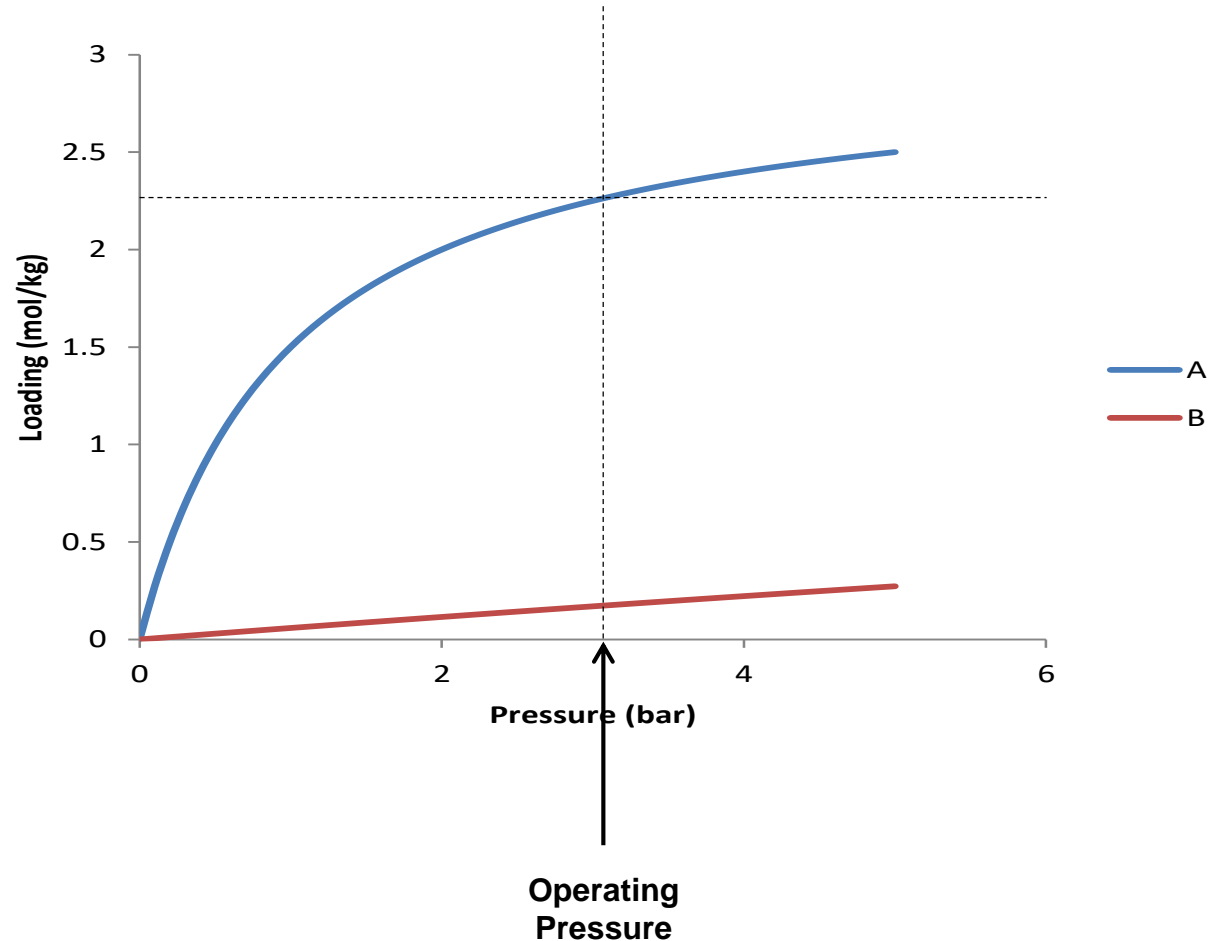


# Methodology



# Pressure Swing Adsorption

How can we obtain B from a mixture of A+B?



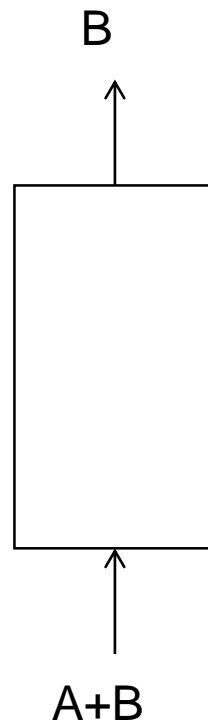
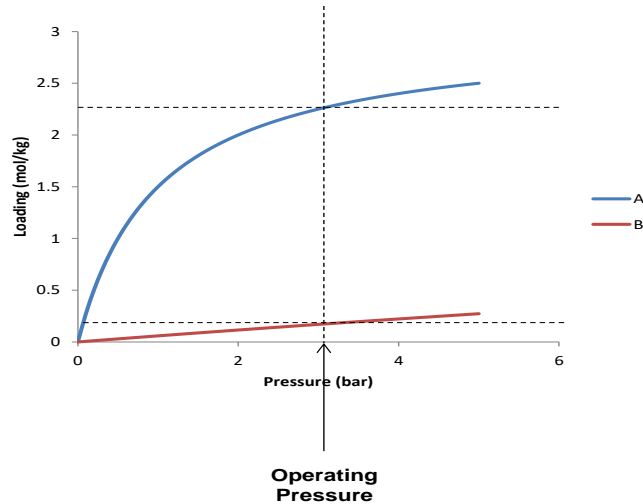
Let's choose an operating pressure considering:

- Acceptable capacity and selectivity;
- Maximum pressure allowed by the adsorbent and unit/equipment;
- Energy consumption;
- Product requirements.



# Pressure Swing Adsorption

How can we obtain B from a mixture of A+B?



If we pass a stream containing A+B at the operating pressures, through a bed packed with this adsorbent:

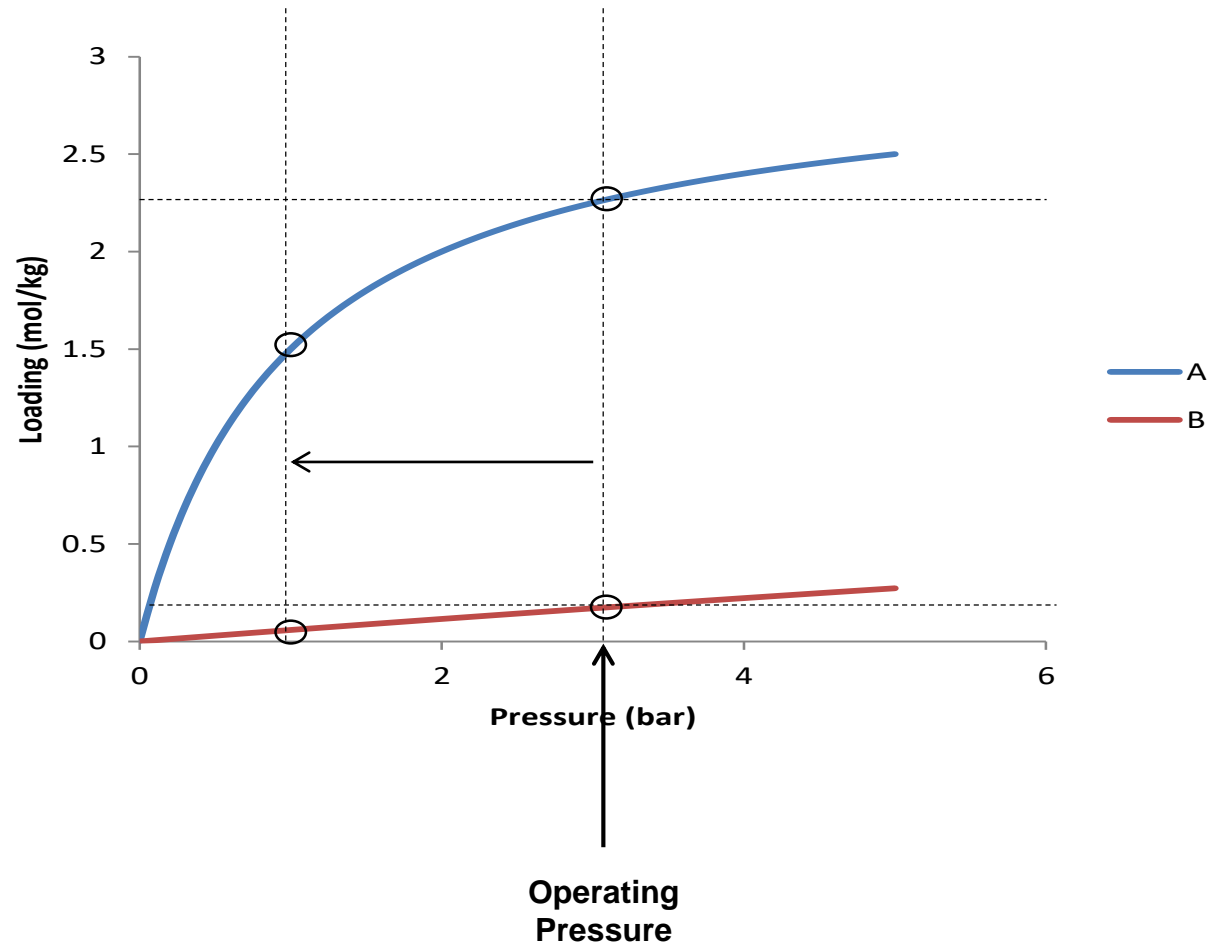
- A will be preferentially adsorbed (retained by the adsorbent);
- B will move through the bed and leave it at the top.

This will happen until the adsorbent is saturated with A.

What should we do next?

# Pressure Swing Adsorption

How can we obtain B from a mixture of A+B?

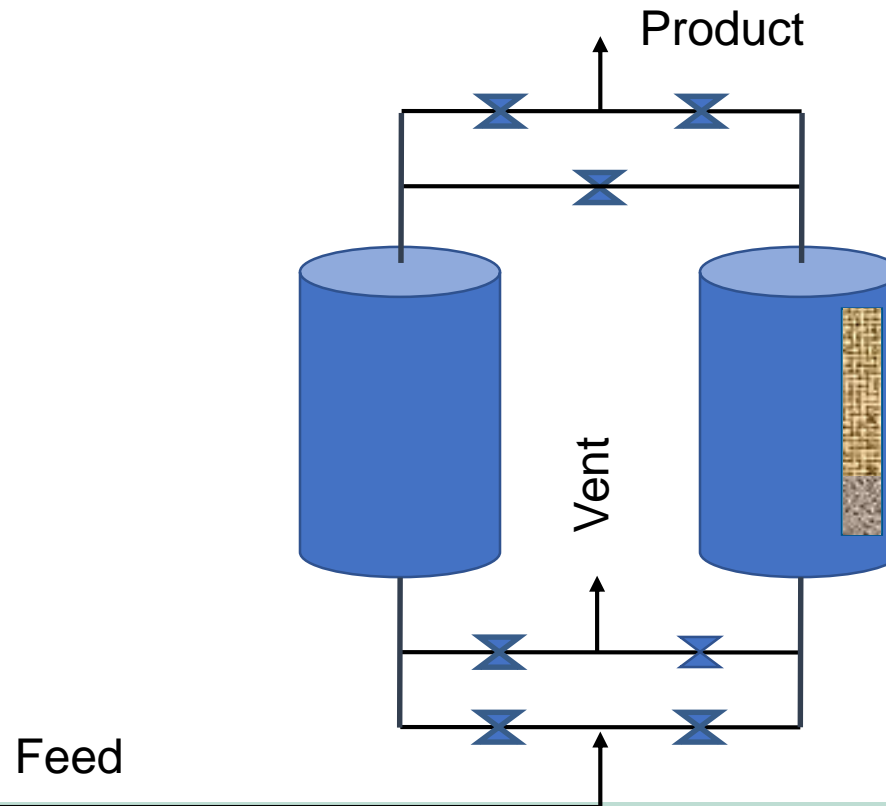


If we decrease the pressure the adsorption capacity also decreases.

For example, if we open the bed to the atmosphere and let the gas out, the final pressure will be 1 bar.

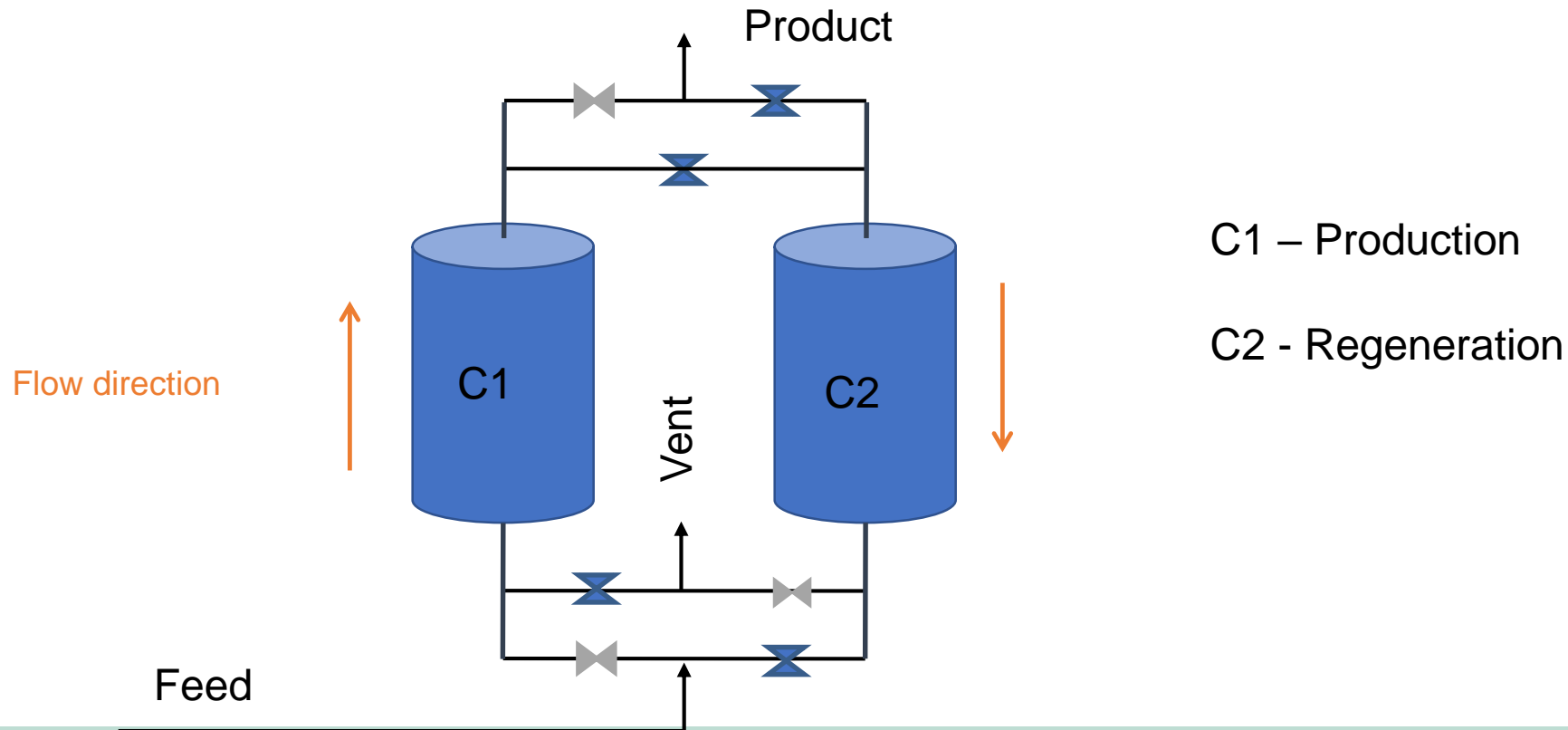
# Pressure Swing Adsorption

PSA units are usually composed by two or more columns packed with one or more adsorbents



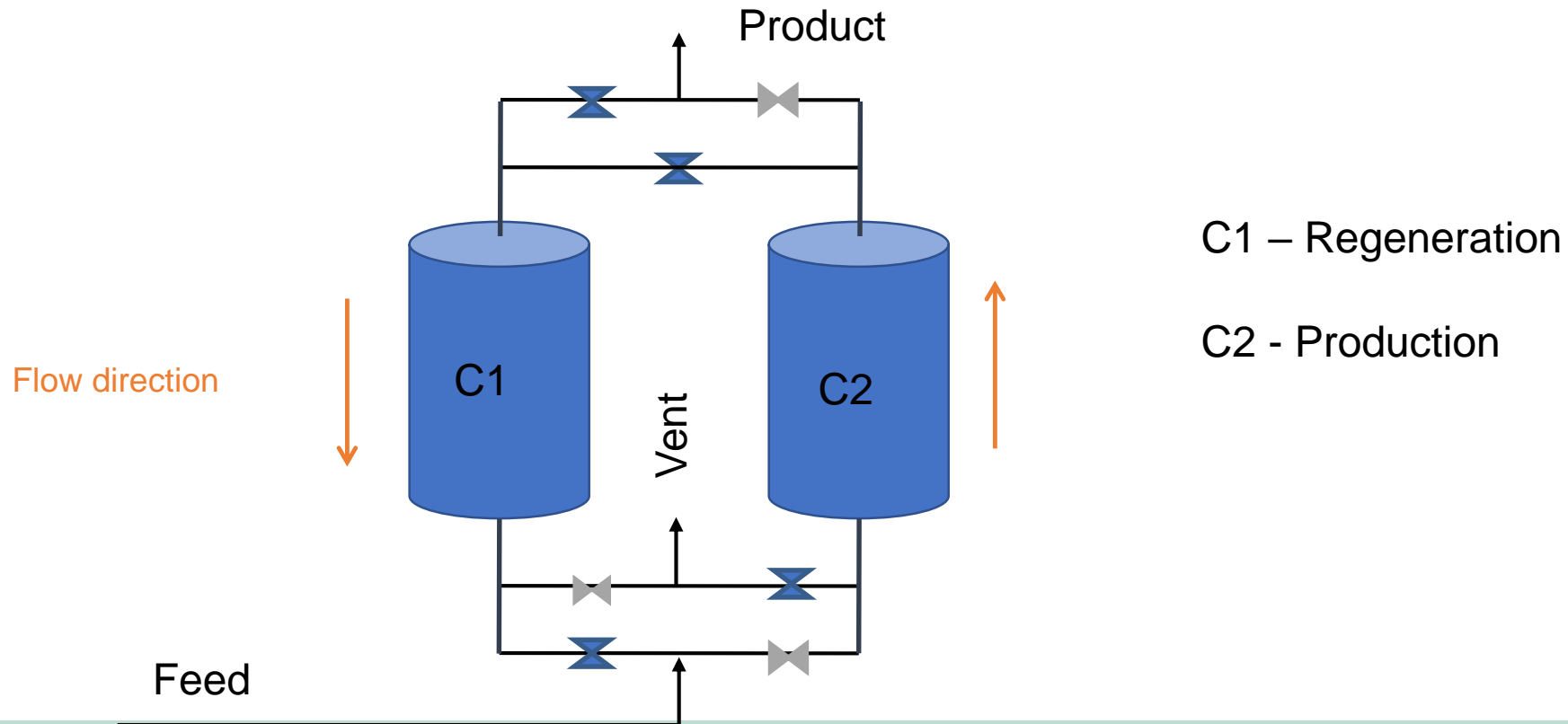
# Pressure Swing Adsorption

PSA units are usually composed by two or more columns packed with one or more adsorbents



# Pressure Swing Adsorption

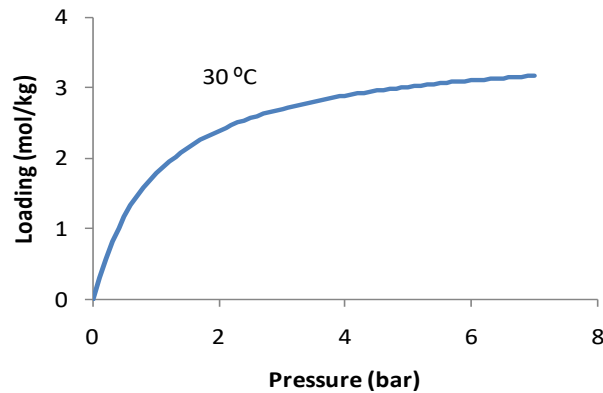
PSA units are usually composed by two or more columns packed with one or more adsorbents



# Adsorbent regeneration - PSA, TSA, CSA

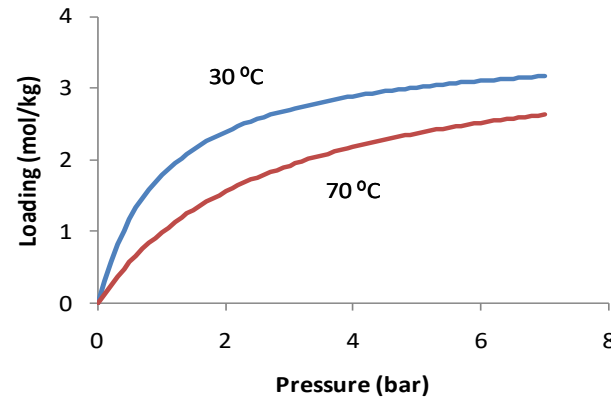
## Pressure Swing Adsorption

The adsorbent is regenerated by lowering the pressure.



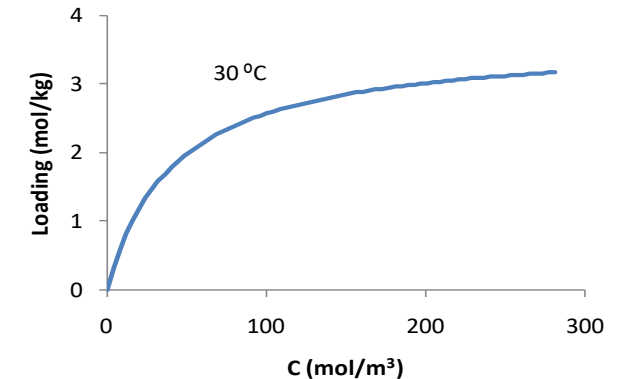
## Temperature Swing Adsorption

The adsorbent is regenerated by increasing the temperature.



## Concentration Swing Adsorption

The adsorbent is regenerated by lowering the concentration.



# SMB principle (Sorbex Processes-UOP)

